

Trapping effects on inflation

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Abstract

We develop a Lagrangian approach based on the influence functional method so as to derive self-consistently the Langevin equation for the inflaton field in the presence of trapping points along the inflaton trajectory. The Langevin equation exhibits the backreaction and the fluctuation-dissipation relation of the trapping. The fluctuation is induced by a multiplicative colored noise that can be identified as the particle number density fluctuations and the dissipation is a new effect that may play a role in the trapping with a strong coupling. In the weak coupling regime, we calculate the power spectrum of the noise-driven inflaton fluctuations for a single trapping point and studied its variation with the trapping location. We also consider a case with closely spaced trapping points and find that the resulting power spectrum is blue.

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I. INTRODUCTION

The inflationary scenario [1], in which the present Universe is only a small local patch of a causally connected region at early times which underwent an exponential expansion driven by the inflaton potential, is generally accepted for explaining the observed spatially flat and homogeneous Universe. In addition, its quantum fluctuations during inflation give rise to primordial Gaussian matter density fluctuations with a nearly scale-invariant power spectrum, which is consistent with recent astrophysical and cosmological observations such as structure formation and cosmic microwave background anisotropies [2].

Although the simplest single-field, slow-roll inflation model works well, some basic questions have yet to be answered. What is the origin of the inflaton potential? Do classical matter density inhomogeneities that we observe today genuinely come from quantum fluctuations of the inflaton? Are the observed matter density fluctuations truly Gaussian? How robust are the predictions for a subdominant contribution of tensor modes to the metric fluctuations, a slightly broken scale invariance, and a negligible running spectral index of the power spectrum? Future cosmic microwave background measurements and mega-scale mappings of the large scale structure will definitely answer some of these questions or perhaps pose a challenge to the standard inflation scenario.

There has been a lot of studies on inflationary models that go beyond the simplest single-field, slow-roll inflation. A class of models has considered a new source for generating inflaton fluctuations during inflation through a Yukawa-type or gravitational interaction between the inflaton and other quantum fields. This leads to very interesting results such as the so-called warm inflation [3], the suppression of large-scale density fluctuations [4], the bursts of particle production via the infra-red cascading mechanism [5], the trapped inflation in which the inflaton rolls slowly down a steep potential by dumping its kinetic energy into particle production [6, 7], and possible constraints on the duration of inflationary expansion [8].

In Refs. [5, 6], the authors analyzed a model in which the inflaton Φ couples to another scalar field χ via the interaction,

$$\frac{g^2}{2}(\Phi - \Phi_0)^2\chi^2, \quad (1)$$

where g is a coupling constant and Φ_0 is a constant field value. When Φ rolls down to Φ_0 , the χ particles become instantaneously massless and are produced with a number density that increases with Φ 's velocity. As Φ dumps its kinetic energy into the χ particles, it is slowed down and the produced χ particles are diluted due to the inflationary expansion. As shown in Ref. [6], a viable inflationary model (the so-called trapped inflation) can be achieved by assuming sufficiently closely spaced trapping points like Φ_0 along the Φ trajectory even on a potential which is too steep for slow-roll inflation. In their approach, the backreaction of the χ particle production to the inflaton field and the associated effect of particle number density fluctuations in the process of particle production are simply introduced in the equation of motion for Φ by use of the mean field approximation:

$$-\nabla_\mu\nabla^\mu\Phi + V'(\Phi) + g^2\langle\chi^2\rangle(\Phi - \Phi_0) = g^2\left(\langle\chi^2\rangle - \chi^2\right)(\Phi - \Phi_0), \quad (2)$$

where $V(\Phi)$ is the inflaton potential and we will go back to this equation later on.

In this paper, we will develop a Lagrangian approach based on the influence functional method in which the χ field is integrated out. This will enable us to derive self-consistently the equation of motion for the Φ field that incorporates the backreaction of the χ particle

production and the particle number density fluctuations in the particle production. We will find that the particle number density fluctuations should be replaced by a colored noise term. In addition, there exists a new dissipative effect that is closely related to the noise. In fact, this dissipation is well-known in the real-time approach to non-equilibrium phenomena in quantum field theory [9, 10]. The paper is organized as follows. In Sec. II, the influence functional method is introduced and the Langevin equation for the inflaton field is derived. In Sec. III, we will calculate the power spectrum of the inflaton fluctuations driven by the noise term in the weak coupling limit. Also, we will consider the effect of multiple trapping points on inflaton fluctuations. The dissipation is discussed in Sec. IV and we conclude in Sec. V.

II. INFLUENCE FUNCTIONAL APPROACH

We will adopt the influence functional method [9] to take into account the effects of the quantum fluctuations of the χ field on the inflaton dynamics in a real-time manner, different from using a one-loop effective potential as usually found for example in Ref. [11]. Thus, the effective Langevin equation of the inflaton is obtained, describing the time-dependent corrections to the inflaton equation of motion originally given by the classical inflaton potential. In particular, this Langevin equation, which goes beyond the mean field approximation, involves a stochastic noise term that drives the growth of perturbation of the inflaton as we will see in the following.

A. Langevin equation for inflaton

Let us consider a slow-rolling inflaton Φ coupled to a massless scalar field χ with an interaction given in Eq.(1). Then, the total Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\Phi) - \frac{g^2}{2}(\Phi - \Phi_0)^2\chi^2, \quad (3)$$

where $V(\Phi)$ is the inflaton potential that complies with the slow-roll conditions. To simplify matter, we make a shift: $\phi = \Phi - \Phi_0$. Hence, the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\phi) - \frac{g^2}{2}\phi^2\chi^2, \quad (4)$$

where the trapping point is located at $\phi = \phi_0 = 0$. We can approximate the space-time during inflation by a de Sitter metric given by

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2), \quad (5)$$

where η is the conformal time and $a(\eta) = -1/(H\eta)$ with H being the Hubble parameter. Here we rescale $a = 1$ at the initial time of the inflation era, $\eta_i = -1/H$.

To proceed, let us assume that the initial density matrix at time η_i can be factorized as

$$\rho(\eta_i) = \rho_\phi(\eta_i) \otimes \rho_\chi(\eta_i). \quad (6)$$

The full density matrix evolves unitarily and the evolution can be described by employing the closed-time-path formalism. Following the influence functional approach [9, 10], we trace

out the field χ in the perturbative expansion. The reduced density matrix of the system then becomes

$$\rho_r(\phi_f, \phi'_f; \eta_f) = \int d\phi_i d\phi'_i \mathcal{Z}(\phi_f, \phi'_f, \eta_f; \phi_i, \phi'_i, \eta_i) \rho_r(\phi_i, \phi'_i; \eta_i). \quad (7)$$

Here the propagating function $\mathcal{Z}(\phi_f, \phi'_f, \eta_f; \phi_i, \phi'_i, \eta_i)$ is obtained as

$$\begin{aligned} \mathcal{Z}(\phi_f, \phi'_f, \eta_f; \phi_i, \phi'_i, \eta_i) &= \int_{\phi_i}^{\phi_f} \mathcal{D}\phi^+ \int_{\phi'_i}^{\phi'_f} \mathcal{D}\phi^- e^{i(S_0[\phi^+] - S_0[\phi^-])} \times e^{iS_{IF}[\phi^+, J^+, \phi^-, J^-]} \\ &= \mathcal{Z}_\phi \cdot \mathcal{Z}_\chi, \end{aligned} \quad (8)$$

with

$$\mathcal{Z}_\chi = e^{iS_{IF}[\phi^+, J^+, \phi^-, J^-]} = \int_{\chi_i}^{\chi_f} \mathcal{D}\chi^+ \int_{\chi'_i}^{\chi'_f} \mathcal{D}\chi^- e^{i(S_0[\chi^+] - S_0[\chi^-])} \times e^{i\frac{g^2}{2} \int d^4x a^4 (\phi^{+2} \chi^{+2} - \phi^{-2} \chi^{-2})}, \quad (9)$$

where the actions for the fields ϕ and χ are given by, respectively,

$$S_0[\phi] = \int d^4x a^2(\eta) \left[\frac{1}{2} \left(\frac{d\phi}{d\eta} \right)^2 - \frac{1}{2} (\nabla\phi)^2 - a^2(\eta) V(\phi) \right], \quad (10)$$

$$S_0[\chi] = \int d^4x a^2(\eta) \left[\frac{1}{2} \left(\frac{d\chi}{d\eta} \right)^2 - \frac{1}{2} (\nabla\chi)^2 \right], \quad (11)$$

and the sources are $J^+ = \phi^{+2}$ and $J^- = \phi^{-2}$. We can expand $e^{iS_{IF}}$ in terms of J^+ and J^- up to g^4 order:

$$\begin{aligned} S_{IF}[\phi^+, J^+, \phi^-, J^-] &= S_{IF}[\phi^+, 0, \phi^-, 0] + \int d^4x a^4(x) \left[\frac{\delta S_{IF}}{\delta J^+} J^+ + \frac{\delta S_{IF}}{\delta J^-} J^- \right] \\ &\quad + \frac{1}{2!} \iint d^4x d^4x' a^4(x) a^4(x') \left[\frac{\delta^2 S_{IF}}{\delta J_x^+ \delta J_{x'}^+} J_x^+ J_{x'}^+ + \frac{\delta^2 S_{IF}}{\delta J_x^+ \delta J_{x'}^-} J_x^+ J_{x'}^- \right. \\ &\quad \left. + \frac{\delta^2 S_{IF}}{\delta J_x^- \delta J_{x'}^+} J_x^- J_{x'}^+ + \frac{\delta^2 S_{IF}}{\delta J_x^- \delta J_{x'}^-} J_x^- J_{x'}^- \right] + \dots, \end{aligned} \quad (12)$$

where the variations with respect to J^+ and J^- are taken at $J^+ = J^- = 0$. The first term $S_{IF}[\phi^+, 0, \phi^-, 0]$ can be absorbed into the normalization constant and the remaining terms are given by

$$\frac{\delta S_{IF}}{\delta J^+} = \left[\frac{-i \delta \mathcal{Z}_\chi}{\mathcal{Z}_\chi \delta J^+} \right]_{J^+=0} = \frac{g^2}{2} \langle \chi^{+2} \rangle, \quad \frac{\delta S_{IF}}{\delta J^-} = \left[\frac{-i \delta \mathcal{Z}_\chi}{\mathcal{Z}_\chi \delta J^-} \right]_{J^-=0} = \frac{-g^2}{2} \langle \chi^{-2} \rangle, \quad (13)$$

$$\begin{aligned} \frac{\delta^2 S_{IF}}{\delta J^+ \delta J^+} &= -i \left[\frac{-1}{\mathcal{Z}_\chi^2} \frac{\delta \mathcal{Z}_\chi}{\delta J^+} \frac{\delta \mathcal{Z}_\chi}{\delta J^+} + \frac{1}{\mathcal{Z}_\chi} \frac{\delta^2 \mathcal{Z}_\chi}{\delta J^+ \delta J^+} \right]_{J^+=0} \\ &= -i \frac{g^4}{4} \langle \chi^{+2}(x) \rangle \langle \chi^{+2}(x') \rangle + i \frac{g^4}{4} \langle \chi^{+2}(x) \chi^{+2}(x') \rangle_{dis}, \end{aligned} \quad (14)$$

$$\begin{aligned}
\frac{\delta^2 S_{IF}}{\delta J^+ \delta J^-} &= -i \left[\frac{-1}{\mathcal{Z}_\chi^2} \frac{\delta \mathcal{Z}_\chi}{\delta J^+} \frac{\delta \mathcal{Z}_\chi}{\delta J^-} + \frac{1}{\mathcal{Z}_\chi} \frac{\delta^2 \mathcal{Z}_\chi}{\delta J^+ \delta J^-} \right]_{J^+=J^-=0} \\
&= i \frac{g^4}{4} \langle \chi^{+2}(x) \rangle \langle \chi^{-2}(x') \rangle - i \frac{g^4}{4} \langle \chi^{+2}(x) \chi^{-2}(x') \rangle_{dis},
\end{aligned} \tag{15}$$

$$\begin{aligned}
\frac{\delta^2 S_{IF}}{\delta J^- \delta J^+} &= -i \left[\frac{-1}{\mathcal{Z}_\chi^2} \frac{\delta \mathcal{Z}_\chi}{\delta J^-} \frac{\delta \mathcal{Z}_\chi}{\delta J^+} + \frac{1}{\mathcal{Z}_\chi} \frac{\delta^2 \mathcal{Z}_\chi}{\delta J^- \delta J^+} \right]_{J^+=J^-=0} \\
&= i \frac{g^4}{4} \langle \chi^{-2}(x) \rangle \langle \chi^{+2}(x') \rangle - i \frac{g^4}{4} \langle \chi^{-2}(x) \chi^{+2}(x') \rangle_{dis},
\end{aligned} \tag{16}$$

$$\begin{aligned}
\frac{\delta^2 S_{IF}}{\delta J^- \delta J^-} &= -i \left[\frac{-1}{\mathcal{Z}_\chi^2} \frac{\delta \mathcal{Z}_\chi}{\delta J^-} \frac{\delta \mathcal{Z}_\chi}{\delta J^-} + \frac{1}{\mathcal{Z}_\chi} \frac{\delta^2 \mathcal{Z}_\chi}{\delta J^- \delta J^-} \right]_{J^-=0} \\
&= -i \frac{g^4}{4} \langle \chi^{-2}(x) \rangle \langle \chi^{-2}(x') \rangle + i \frac{g^4}{4} \langle \chi^{-2}(x) \chi^{-2}(x') \rangle_{dis},
\end{aligned} \tag{17}$$

where the subscript *dis* means disconnected diagrams. The disconnected diagrams can be contracted to the connected diagrams as the following:

$$\langle \chi^{+2}(x) \chi^{+2}(x') \rangle_{dis} = 2 \langle \chi^+(x) \chi^+(x') \rangle^2 + \langle \chi^{+2}(x) \rangle \cdot \langle \chi^{+2}(x') \rangle, \tag{18}$$

$$\langle \chi^{+2}(x) \chi^{-2}(x') \rangle_{dis} = 2 \langle \chi^+(x) \chi^-(x') \rangle^2 + \langle \chi^{+2}(x) \rangle \cdot \langle \chi^{-2}(x') \rangle, \tag{19}$$

$$\langle \chi^{-2}(x) \chi^{+2}(x') \rangle_{dis} = 2 \langle \chi^-(x) \chi^+(x') \rangle^2 + \langle \chi^{-2}(x) \rangle \cdot \langle \chi^{+2}(x') \rangle, \tag{20}$$

$$\langle \chi^{-2}(x) \chi^{-2}(x') \rangle_{dis} = 2 \langle \chi^-(x) \chi^-(x') \rangle^2 + \langle \chi^{-2}(x) \rangle \cdot \langle \chi^{-2}(x') \rangle. \tag{21}$$

After contraction, we are able to obtain the influence functional up to order g^4 as

$$\begin{aligned}
e^{iS_{IF}[\phi^+, \phi^-]} &= \exp \left\{ i \frac{g^2}{2} \int d^4 x_1 a^4(\eta_1) \left[\phi^{+2}(x_1) \langle \chi^+(x_1) \chi^+(x_1) \rangle - \phi^{-2}(x_1) \langle \chi^-(x_1) \chi^-(x_1) \rangle \right] \right. \\
&\quad - \frac{g^4}{4} \int d^4 x_1 \int d^4 x_2 a^4(\eta_1) a^4(\eta_2) \\
&\quad \left[\phi^{+2}(x_1) \langle \chi^+(x_1) \chi^+(x_2) \rangle^2 \phi^{+2}(x_2) - \phi^{+2}(x_1) \langle \chi^+(x_1) \chi^-(x_2) \rangle^2 \phi^{-2}(x_2) \right. \\
&\quad \left. \left. - \phi^{-2}(x_1) \langle \chi^-(x_1) \chi^+(x_2) \rangle^2 \phi^{+2}(x_2) + \phi^{-2}(x_1) \langle \chi^-(x_1) \chi^-(x_2) \rangle^2 \phi^{-2}(x_2) \right] \right\} \tag{22}
\end{aligned}$$

The Green's functions of the χ field are defined by

$$\begin{aligned}
\langle \chi^+(x) \chi^+(x') \rangle &= \langle \chi(x) \chi(x') \rangle \theta(\eta - \eta') + \langle \chi(x') \chi(x) \rangle \theta(\eta' - \eta), \\
\langle \chi^-(x) \chi^-(x') \rangle &= \langle \chi(x') \chi(x) \rangle \theta(\eta - \eta') + \langle \chi(x) \chi(x') \rangle \theta(\eta' - \eta), \\
\langle \chi^+(x) \chi^-(x') \rangle &= \langle \chi(x) \chi(x') \rangle, \\
\langle \chi^-(x) \chi^+(x') \rangle &= \langle \chi(x') \chi(x) \rangle,
\end{aligned} \tag{23}$$

and can be explicitly constructed as long as its vacuum state has been specified. To obtain the semiclassical Langevin equation, it is more convenient to introduce the average and relative field variables:

$$\phi = \frac{1}{2}(\phi^+ + \phi^-), \quad \phi_\Delta = \phi^+ - \phi^-. \tag{24}$$

The coarse-grained effective action (CGEA) including the influence action S_{IF} obtained from Eqs. (8) to (22) is then given by

$$\begin{aligned}
S_{CGEA}[\phi, \phi_\Delta] = & \int d^4x a^2(\eta) \phi_\Delta(x) \left\{ -\ddot{\phi}(x) - 2aH\dot{\phi}(x) + \nabla^2\phi(x) - a^2 \left[V'(\phi) + g^2\langle\chi^2\rangle\phi(x) \right] \right. \\
& \left. - g^4 a^2(\eta)\phi(x) \int d^4x' a^4(\eta') \theta(\eta - \eta') iG_-(x, x')\phi^2(x') \right\} \\
& + i\frac{g^4}{2} \int d^4x \int d^4x' a^4(\eta)a^4(\eta') \phi_\Delta(x)\phi(x) G_+(x, x') \phi_\Delta(x')\phi(x') + \mathcal{O}(\phi_\Delta^3),
\end{aligned} \tag{25}$$

where the dot and prime denote respectively differentiation with respect to η and ϕ . In addition, we have used the fact that correlation functions of the fields evaluated at the same space-time point in the $+$ and $-$ branches are equal, namely, $\langle\chi^+(x_1)\chi^+(x_1)\rangle = \langle\chi^-(x_1)\chi^-(x_1)\rangle \equiv \langle\chi^2(x_1)\rangle$. The kernels G_\pm can be obtained from the Green's function of χ :

$$G_+(x, x') = \langle\chi(x)\chi(x')\rangle^2 + \langle\chi(x')\chi(x)\rangle^2, \tag{26}$$

$$G_-(x, x') = \langle\chi(x)\chi(x')\rangle^2 - \langle\chi(x')\chi(x)\rangle^2. \tag{27}$$

The imaginary part of the above influence action can be re-expressed by introducing an auxiliary field ξ with a distribution function of the Gaussian form,

$$P[\xi] = \exp \left\{ -\frac{1}{2} \int d^4x \int d^4x' \xi(x) \nu^{-1}(x, x') \xi(x') \right\}, \tag{28}$$

where the noise kernel is

$$\nu(x, x') = \langle\xi(x)\xi(x')\rangle = G_+(x, x'). \tag{29}$$

This leads to

$$e^{iS_{CGEA}} = \int \mathcal{D}\xi P[\xi] \exp iS_{\text{eff}}[\phi, \phi_\Delta, \xi], \tag{30}$$

with the effective action S_{eff} given by

$$\begin{aligned}
S_{\text{eff}}[\phi, \phi_\Delta, \xi] = & \int d^4x a^2(\eta) \phi_\Delta(x) \left\{ -\ddot{\phi}(x) - 2aH\dot{\phi}(x) + \nabla^2\phi(x) - a^2 \left[V'(\phi) + g^2\langle\chi^2\rangle\phi(x) \right] \right. \\
& \left. - g^4 a^2\phi(x) \int d^4x' a^4(\eta') \theta(\eta - \eta') iG_-(x, x')\phi^2(x') + g^2 a^2\phi(x)\xi(x) \right\}.
\end{aligned} \tag{31}$$

The semiclassical approximation requires to extremize the effective action $\delta S_{\text{eff}}/\delta\phi_\Delta$ when long-wavelength inflaton modes of cosmological interest have gone through the quantum-to-classical transition due to the rapid expansion of the scale factor [12]. Then, we obtain the semiclassical Langevin equation for ϕ :

$$\begin{aligned}
& \ddot{\phi} + 2aH\dot{\phi} - \nabla^2\phi + a^2 \left[V'(\phi) + g^2\langle\chi^2\rangle\phi \right] + g^4 a^2\phi \int d^4x' a^4(\eta') \times \\
& \theta(\eta - \eta') iG_-(x, x')\phi^2(x') = g^2 a^2\phi\xi.
\end{aligned} \tag{32}$$

This is the main result of our paper. It shows that the effects from the quantum field χ on the inflaton are given by the dissipation via the kernel G_- as well as the fluctuation induced by the multiplicative colored noise ξ with

$$\langle\xi(x)\xi(x')\rangle = G_+(x, x'). \tag{33}$$

B. Approximate solutions

To solve Eq. (32), let us first drop the dissipative term which we will discuss later and consider the colored noise only. Then, Eq. (32) becomes

$$\ddot{\phi} + 2aH\dot{\phi} - \nabla^2\phi + a^2 [V'(\phi) + g^2\langle\chi^2\rangle\phi] = g^2 a^2 \phi \xi. \quad (34)$$

After decomposing ϕ into a mean field and a classical perturbation: $\phi(\eta, \mathbf{x}) = \bar{\phi}(\eta) + \varphi(\eta, \mathbf{x})$, we obtain the linearized Langevin equation,

$$\ddot{\varphi} + 2aH\dot{\varphi} - \nabla^2\varphi + a^2 m_{\varphi\text{eff}}^2 \varphi = g^2 a^2 \bar{\phi} \xi, \quad (35)$$

where the effective mass is $m_{\varphi\text{eff}}^2 = V''(\bar{\phi}) + g^2\langle\chi^2\rangle$ and the time evolution of $\bar{\phi}$ is governed by

$$\ddot{\bar{\phi}} + 2aH\dot{\bar{\phi}} + a^2 [V'(\bar{\phi}) + g^2\langle\chi^2\rangle\bar{\phi}] = 0. \quad (36)$$

The equation of motion for χ from which we construct its Green's function can be read off from its quadratic terms in the Lagrangian (4) as

$$\ddot{\chi} + 2aH\dot{\chi} - \nabla^2\chi + a^2 m_{\chi\text{eff}}^2 \chi = 0, \quad (37)$$

where the effective mass is $m_{\chi\text{eff}}^2 = g^2\bar{\phi}^2$. Let us decompose

$$\begin{aligned} Y(x) &= \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} Y_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad \text{where } Y = \varphi, \xi, \\ \chi(x) &= \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} [b_{\mathbf{k}}\chi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.}], \end{aligned} \quad (38)$$

where $b_{\mathbf{k}}^\dagger$ and $b_{\mathbf{k}}$ are creation and annihilation operators satisfying $[b_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')$. Then, the solution to Eq. (35) is obtained as

$$\varphi_{\mathbf{k}}(\eta) = g^2 \int_{\eta_i}^{\eta} d\eta' a^2(\eta') \bar{\phi}(\eta') \xi_{\mathbf{k}}(\eta') G_r(\eta', \eta), \quad (39)$$

where we have adopted the retarded Green's function and

$$G_r(\eta', \eta) = [\varphi_k^1(\eta')\varphi_k^2(\eta) - \varphi_k^2(\eta')\varphi_k^1(\eta)] W^{-1} [\varphi_k^1(\eta'), \varphi_k^2(\eta')]. \quad (40)$$

Here the homogeneous solutions $\varphi_k^{1,2}$ of Eq. (35) are given by

$$\varphi_k^{1,2} = \frac{1}{2a} (\pi|\eta|)^{\frac{1}{2}} H_{\nu}^{(1),(2)}(k\eta) \quad (41)$$

and the Wronskian is

$$W [\varphi_k^1(\eta), \varphi_k^2(\eta)] = \varphi_k^1(\eta) \frac{d\varphi_k^2(\eta)}{d\eta} - \varphi_k^2(\eta) \frac{d\varphi_k^1(\eta)}{d\eta} = \frac{i}{a^2(\eta)}. \quad (42)$$

$H_{\nu}^{(1)}$ and $H_{\nu}^{(2)}$ are Hankel functions of the first and second kinds respectively and $\nu^2 = 9/4 - m_{\varphi\text{eff}}^2/H^2$. In addition, we have from Eq. (37) that

$$\chi_k(\eta) = \frac{1}{2a} (\pi|\eta|)^{\frac{1}{2}} [c_1 H_{\mu}^{(1)}(k\eta) + c_2 H_{\mu}^{(2)}(k\eta)], \quad (43)$$

where the constants c_1 and c_2 are subject to the normalization condition, $|c_2|^2 - |c_1|^2 = 1$, and $\mu^2 = 9/4 - m_{\chi\text{eff}}^2/H^2$.

III. TRAPPING EFFECTS TO INFLATION

The Langevin equation (34) that we have derived by going beyond the mean field approximation, though similar to the equation of motion (2) used in Refs. [5, 6], has more physical meanings. While the backreaction is a common feature, the particle number density fluctuations in Eq. (2) are replaced by a colored noise (33) that appears as a source term in Eq. (34). In addition, the full Langevin equation (32) has a new dissipation term that is expected to co-exist with the noise term by virtue of the fluctuation-dissipation theorem [10].

Now we are ready to calculate the power spectrum of the perturbation φ induced by the noise term. We already have $V''(\bar{\phi}) \ll H^2$ for a slow-roll inflaton potential. In order to maintain the slow-roll condition: $m_{\phi\text{eff}}^2 = m_{\varphi\text{eff}}^2 \ll H^2$ (i.e., $\nu = 3/2$), we further require that $g^2 \langle \chi^2 \rangle \ll H^2$.

A. Weak coupling limit

We consider, as the simplest case, a very weak coupling constant, $g^2 \ll 1$. This allows us to highlight the effect of trapping to inflation, although the effect is too small to be observed. In the weak coupling limit, we simply have $\nu = \mu = 3/2$. It was shown that when $\mu = 3/2$ one can select the Bunch-Davies vacuum (i.e., $c_2 = 1$ and $c_1 = 0$) in Eq. (43) [14]. Hence, using Eqs. (33) and (39), we obtain

$$\langle \varphi_{\mathbf{k}}(\eta) \varphi_{\mathbf{k}'}^*(\eta) \rangle = \frac{2\pi^2}{k^3} \Delta_k^\xi(\eta) \delta(\mathbf{k} - \mathbf{k}'), \quad (44)$$

where the noise-driven power spectrum is given by

$$\Delta_k^\xi(\eta) = \frac{g^4 z_-^2}{8\pi^4} \int_{z_i}^z dz_1 \int_{z_i}^z dz_2 \bar{\phi}(\eta_1) \bar{\phi}(\eta_2) F(z_1) F(z_2) \left\{ \frac{\sin z_-}{z_1 z_2 z_-} [\sin(2\Lambda z_-/k)/z_- - 1] + G(z_1, z_2) \right\}, \quad (45)$$

where $z_- = z_2 - z_1$, $z = k\eta$, $z_i = k\eta_i = -k/H$, Λ is the momentum cutoff introduced in the evaluation of the ultraviolet divergent k -integration of χ_k in the Green's function (26),

$$F(y) = \left(1 + \frac{1}{yz}\right) \sin(y - z) + \left(\frac{1}{y} - \frac{1}{z}\right) \cos(y - z), \quad (46)$$

and

$$\begin{aligned} & G(z_1, z_2) \\ &= \int_0^\Lambda dk_1 \int_{|k-k_1|}^{k+k_1} dq \left\{ \left[\frac{2}{z_1 z_2 q k_1} \left(\frac{k_1}{q} - \frac{z_1}{z_2} - \frac{z_2}{z_1} + 2 + \frac{k^2}{z_1 z_2 k_1 q} \right) \right] \cos \left[\left(\frac{k_1 + q}{k} \right) (z_2 - z_1) \right] \right. \\ &+ \frac{2}{kq} \left(\frac{1}{z_1} - \frac{1}{z_2} \right) \left[1 + \frac{k^2}{z_1 z_2 k_1} \left(\frac{1}{q} + \frac{1}{k_1} \right) \right] \sin \left[\left(\frac{k_1 + q}{k} \right) (z_2 - z_1) \right] \Big\} \\ &+ \frac{2}{z_1 z_2} \int_0^\Lambda \frac{dk_1}{k_1} \left\{ \cos(z_2 - z_1) - \cos \left[(z_2 - z_1) \left(\frac{2k_1}{k} - 1 \right) \right] \right\} \\ &+ \frac{2k}{z_1 z_2 (z_2 - z_1)} \left\{ \int_0^\Lambda \frac{dk_1}{k_1^2} \sin \left[(z_2 - z_1) \left(1 + \frac{2k_1}{k} \right) \right] \right. \\ &- \int_0^k \frac{dk_1}{k_1^2} \sin(z_2 - z_1) - \int_k^\Lambda \frac{dk_1}{k_1^2} \sin \left[(z_2 - z_1) \left(\frac{2k_1}{k} - 1 \right) \right] \Big\}. \end{aligned} \quad (47)$$

Note that the term $\sin(2\Lambda z_-/k)/z_- \simeq \pi\delta(z_-)$ in Eq. (45) when $\Lambda \gg k$. Now we can approximate $\bar{\phi}(\eta)$ by

$$\bar{\phi}(\eta) = v(t_0 - t) = \frac{v}{H} \ln \frac{\eta}{\eta_0}, \quad (48)$$

where the real time t is defined by $a(\eta) = a(t) = e^{Ht}$, t_0 is the time when $\bar{\phi}$ reaches the trapping point, and v is the slow-roll velocity of $\bar{\phi}$. Given the inflaton potential $V(\bar{\phi})$, $v \simeq -V'(\bar{\phi})/(3H)$. After substituting Eq. (48) for $\bar{\phi}(\eta_1)$ and $\bar{\phi}(\eta_2)$ in Eq. (45), we numerically evaluate the multiple integral. In Fig. 1, we plot $\Delta_k^\xi(\eta)$ evaluated at the horizon-crossing time, which is given by $z = -2\pi$, versus k/H for various locations of the trapping point along the inflaton trajectory. Using the e-folding Ht_0 to mark the moment when $\bar{\phi}$ hits the trapping point, we investigate the effect of trapping by putting the trapping point at $Ht_0 = 2, 4, 10$. In the figure, the modes with $k/H = 2\pi$ and $k/H = 500$ cross out the horizon at the start of inflation and at about 4.4 e-folds, respectively. The figure shows the dependence of the noise-driven fluctuations on the onset time of inflation, reflecting the integrated effect of the source powered by the noise term. In the case with $Ht_0 = 2$, the multiplicative noise $\bar{\phi}\xi$ in Eq. (35) vanishes at the trapping point before the mode with $k/H = 500$ leaves the horizon, resulting in a dip in the power spectrum. When Ht_0 increases, the trapping effect becomes insignificant for the range of k -mode in the figure. In evaluating the integral in Eq. (47), we have set the cutoff scale $\Lambda = 200\pi$ to obtain the power spectra and found that the results are insensitive to the choice of the cutoff value.

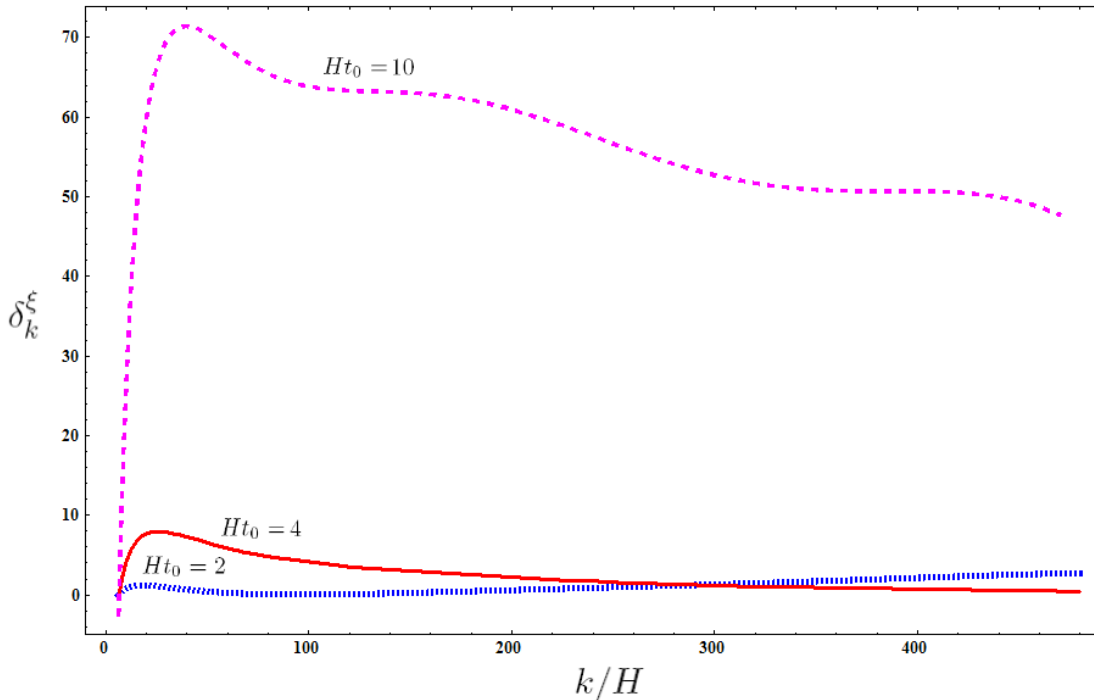


FIG. 1: Power spectra of the noise-driven inflaton fluctuations $\delta_k^\xi \equiv 2\pi^2 H^2 \Delta_k^\xi / (g^4 v^2) = (2\pi^2/g^4)(2\pi/H)^2 P_\zeta \Delta_k^\xi$, where P_ζ is defined in Eq. (50), with the trapping points located at $Ht_0 = 2, 4, 10$ respectively. The starting point, $k/H = 2\pi$, corresponds to the k -mode that leaves the horizon at the start of inflation.

B. Closely spaced trapping points

It is useful to define a time scale, $\Delta t \equiv 1/\sqrt{g\bar{v}}$. Then, We have

$$H\Delta t = \left(\frac{2\pi}{g}\right)^{\frac{1}{2}} P_{\zeta}^{\frac{1}{4}}, \quad (49)$$

where P_{ζ} is the matter power spectrum given by

$$P_{\zeta} = \left(\frac{H}{v}\right)^2 \left(\frac{H}{2\pi}\right)^2. \quad (50)$$

It has been measured by the seven-year WMAP to be $P_{\zeta} \simeq 2.4 \times 10^{-9}$ [13].

The χ particle production can be obtained by solving Eq. (37), in which the effective mass is

$$m_{\chi\text{eff}}^2 = g^2\bar{\phi}^2 = g^2v^2(t_0 - t)^2 = \Delta t^{-4}(t_0 - t)^2. \quad (51)$$

When $t \simeq t_0$, the χ particles become instantaneously massless and particle production begins. For $H\Delta t < 1$ or $g^2 > 10^{-7}$, it was shown [5, 6] that bursts of particle production takes place in a time scale, Δt , and the number density of the χ particles produced is given by $n_{\chi} \simeq 1/\Delta t^3$. This leads to the backreaction of the χ particle production to the inflaton field and the associated effect of particle number density fluctuations in the process of particle production. Furthermore, for closely spaced trapping points with equal spacing Γ along the inflaton trajectory, trapped inflation occurs even on a potential which is too steep for slow-roll inflation, provided that $H\Gamma/v \ll 1$ [6].

Here we consider the case for $H\Delta t \simeq 1$ or $g^2 \simeq 10^{-7}$. It is well-known [14] that for a massless field, the quantum fluctuations grow linearly with time: $\langle\chi^2\rangle \simeq H^3t/(4\pi^2)$. As $\bar{\phi}$ is moving away from the trapping point, χ acquires an effective mass and the growth slows down. This happens in a time scale of Δt , so $n_{\chi} = m_{\chi\text{eff}}\langle\chi^2\rangle/2 = H^3/(8\pi^2)$. This is consistent with the estimate in the above paragraph. Also, $g^2\langle\chi^2\rangle \simeq 10^{-9}H^2$, which satisfies the slow-roll requirement. Therefore, within the time scale Δt , both $m_{\chi\text{eff}}^2$ and $m_{\phi\text{eff}}^2$ are smaller than H^2 and we can approximate the homogeneous φ solution and the χ solution by the mode functions in Eq. (39) with $\nu = 3/2$ and Eq. (43) with $\mu = 3/2$, respectively.

Assume that there exist evenly spaced trapping points with $\Gamma/v < \Delta t$ along the inflaton trajectory. For each trapping event, $\bar{\phi}(\eta')$ in Eq. (39) can be approximately replaced by the constant spacing Γ and

$$G_r(\eta', \eta) = G_r(\eta', \eta' + \Delta\eta') \simeq \Delta\eta' = \frac{\Gamma}{v} \frac{1}{a(\eta')}. \quad (52)$$

Then, we can estimate the noise-driven inflaton fluctuations in this single trapping as

$$d\varphi_{\mathbf{k}}(\eta') \simeq \frac{g^2\Gamma^2}{v} d\eta' a(\eta') \xi_{\mathbf{k}}(\eta'). \quad (53)$$

Hence, at time η the accumulative noise-driven inflaton fluctuations are given by

$$\varphi_{\mathbf{k}}(\eta) \simeq \frac{g^2\Gamma^2}{v} \int_{\eta_i}^{\eta} d\eta' a(\eta') \xi_{\mathbf{k}}(\eta'), \quad (54)$$

and the power spectrum is

$$\Delta_k^\xi(\eta) = \frac{g^4 \Gamma^4 H^2}{8\pi^4 v^2} \int_{z_i}^z dz_1 \int_{z_i}^z dz_2 z_1 z_2 \left\{ \frac{\sin z_-}{z_-} [\sin(2\Lambda z_-/k)/z_- - 1] + G(z_1, z_2) \right\}. \quad (55)$$

We plot this $\Delta_k^\xi(\eta)$ at the horizon-crossing time given by $z = -2\pi$ versus k/H in Fig. 2. The figure shows that the total noise-driven fluctuations due to closely spaced trapping points depend on the onset time of inflation and have a blue power spectrum Δ_k^ξ that increases with k . When Γ saturates the upper limit, i.e. $\Gamma = v\Delta t$, we have $\Gamma = 1/(g\Delta t) \simeq H/g$ and Δ_k^ξ increases to about $10^6 g^2 H^2/(8\pi^4)$ at $k/H = 300$, which is still much smaller than the intrinsic de Sitter quantum fluctuations by a factor of about $4\pi^2/(10^6 g^2) \simeq 400$.

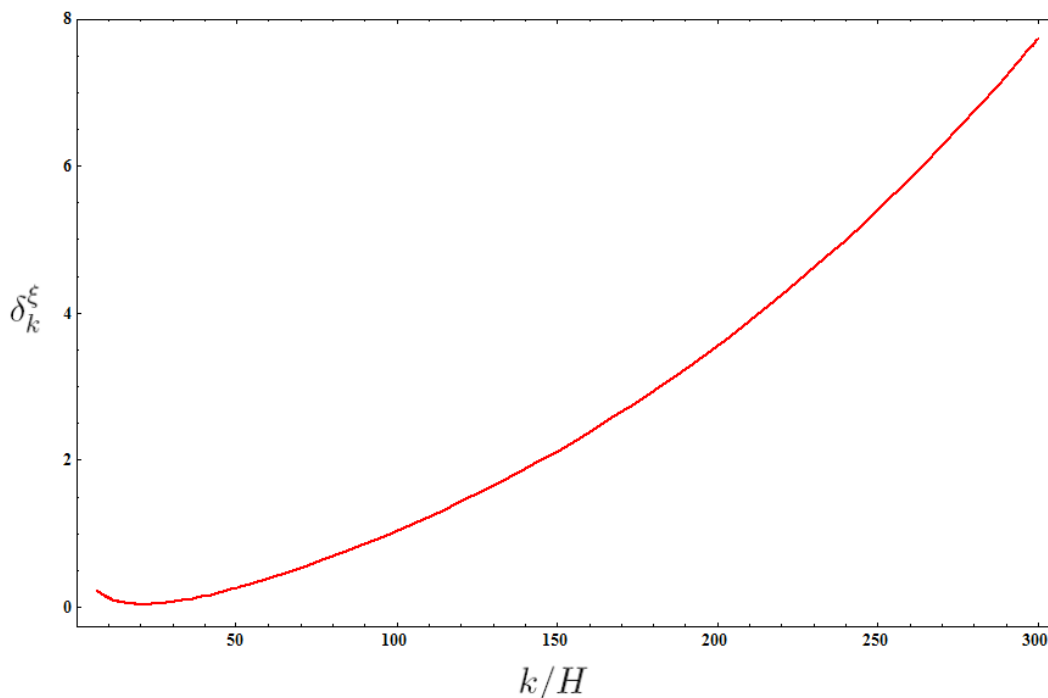


FIG. 2: Accumulative power spectrum of the noise-driven inflaton fluctuations $\delta_k^\xi \equiv 10^{-5} 8\pi^4 v^2 \Delta_k^\xi / (g^4 \Gamma^4 H^2)$ for the case with closely spaced trapping points with even spacing Γ along the inflaton trajectory.

IV. DISSIPATION TERM

The dissipation term in the Langevin equation (32) for inflaton is of order g^4 . In Ref. [4], a model that has the Lagrangian (4) plus a mass term $H^2 \chi^2/2$ for the χ field was considered. It was shown that including the dissipation term would only slightly affect the kinematics for slow-roll inflation as well as the noise-driven inflaton fluctuations even with $g^2 \sim 1$. In this paper, we have considered a weak coupling constant with $g^2 < 10^{-7}$. Although the χ field is massless here, the results for the noise-driven inflaton fluctuations are similar to those in

Ref. [4]. Therefore, we expect that the dissipation term can be safely omitted. However, for strong couplings with $g^2 > 10^{-7}$, such as those considered in the study of particle bursts [5] and in the trapped inflation [6], the dissipation term may be in competition with the noise term. If so, one would need to solve the full Langevin equation (32) to re-examine the inflation dynamics and the inflaton fluctuations.

V. CONCLUSIONS

We have developed a Lagrangian approach based on the influence functional method to investigate the effects of trapping on inflation. The Langevin equation for inflaton thus derived in Eq. (32) is compared to the equation of motion used in Refs. [5, 6]. The multiplicative colored noise in the Langevin equation is indeed the particle number density fluctuations studied by the authors in Refs. [5, 6]; however, they have not considered the dissipation term.

The Langevin equation has been solved in the weak coupling regime with $g^2 < 10^{-7}$, in which the dissipation can be ignored. We have calculated the power spectrum of the noise-driven inflaton fluctuations for a single trapping point and studied its variation with the location of the trapping point along the inflaton trajectory. We have found that if the inflaton rolls down the potential with closely spaced trapping points, the resulting power spectrum would be blue. This is an interesting result. However, the dissipation should begin to play a role and damp the power of the high k modes which leave the horizon in late times.

The present paper has given a systematic approach to deal with trapping points located along the inflaton trajectory. It warrants re-examining the effects of trapping with a strong coupling on inflation that have been discussed in Refs. [5, 6]. In particular, it is interesting to study the effect of dissipation to both the backreaction and the fluctuations.

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